faculty of science
department of mathematics

Midterm I

MATH 232 D100 Spring 2012

Instructor: D. J. Katz

February 1, 2012, 11:30 a.m. – 12:20 p.m.

Name: ______________________________ (please print)
family name *given name*

SFU ID: ______________________________ @sfu.ca
student number *SFU-email*

Signature: ______________________________

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters. Write your SFU student number and email ID on the line provided for it.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. To receive full credit for a particular question you must provide a complete and well presented solution.
5. This exam has 5 questions on 5 pages (not including this cover page). Once the exam begins please check to make sure your exam is complete.
6. **No** calculators, books, papers, or electronic devices shall be within the reach of a student during the examination. Leave answers in "calculator ready" expressions: such as $3 + \ln 7$ or $e^{\sqrt{2}}$.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, or copying from, other examinees is forbidden.**

Question	Maximum	Score
1	8	
2	11	
3	18	
4	10	
5	12	
Total	59	

1. Let $A = \begin{pmatrix} 1 & 2 \\ -3 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 2 & 3 \end{pmatrix}$, $\mathbf{u} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$, and $\mathbf{v} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$.

[2] (a) Compute AB.

[2] (b) Compute B^T.

[2] (c) Compute A^{-1}.

[2] (d) Compute $\mathbf{u}\mathbf{v}^T$.

2. Let $\mathbf{u} = (1, 2, -1)$ and $\mathbf{v} = (3, -2, -1)$.

[2] (a) Show that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal with a brief calculation.

[2] (b) Set up, but do not solve, a linear system in matrix form $A\mathbf{x} = \mathbf{b}$ whose solutions are precisely the vectors $\mathbf{x}$ that make $\{\mathbf{u}, \mathbf{v}, \mathbf{x}\}$ an orthogonal set.

[4] (c) Which of the following vectors $\mathbf{w}_j$ makes $\{\mathbf{u}, \mathbf{v}, \mathbf{w}_j\}$ an orthogonal set?

$$\mathbf{w}_1 = (4, 0, 4), \qquad \mathbf{w}_2 = (2, 1, 4), \qquad \mathbf{w}_3 = (0, 2, -4)$$

For each vector you must either show why it does make the set orthogonal or why it does not.

[3] (d) Find an orthonormal set of three vectors in $\mathbb{R}^3$ containing a vector parallel to $\mathbf{u}$, a vector parallel to $\mathbf{v}$, and one other vector.

3. For each question, circle the correct answer, either "True" or "False." **For each question, you get 3 points for the right answer, 0 points for the wrong one, and 1.5 points for leaving it blank. You do not need to justify your answers.**

[3] (a) If A is a square matrix whose columns form an orthonormal set, then A must be invertible.

True **False**

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[3] (b) If A and B are square matrices of the same size, then $\mathrm{tr}(AB) = \mathrm{tr}(A)\,\mathrm{tr}(B)$.

True **False**

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[3] (c) A consistent linear system with two equations in three unknowns can not have only one solution.

True **False**

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[3] (d) If A is a square matrix, and the linear systems $A\mathbf{x} = \mathbf{0}$ and $A\mathbf{x} = \mathbf{b}$ have different numbers of solutions, then A must not be invertible.

True **False**

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[3] (e) A matrix with linearly independent columns must have linearly independent rows.

True **False**

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[3] (f) If k is a positive integer and the span of $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ is the same as the span of $T = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{k-1}\}$, then S must be linearly dependent.

True **False**

[10] **4.** Consider the linear system

$$\begin{pmatrix} 2 & -2 & 3a+2 \\ 1 & a & 1 \\ 1 & -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ a \end{pmatrix}$$

Determine the value(s) of a, if any, for which this system has no solutions.
Determine the value(s) of a, if any, for which this system has precisely one solution.
Determine the value(s) of a, if any, for which this system has infinitely many solutions.

5. Let $P = (-1, 3, -2)$ and $Q = (1, -1, 4)$ be points in $\mathbb{R}^3$. Let L be the plane consisting of all the points (x, y, z) such that the distance between (x, y, z) and P is the same as the distance between (x, y, z) and Q.

[6] (a) Show that the general equation of the plane L is $x - 2y + 3z = 1$.

[6] (b) Give a either a vector equation or a set of parametric equations for the intersection between the plane L and the plane given by $-x + 3y - 2z = 0$.